OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4766/01**

MATHEMATICS (MEI)

Statistics 1

FRIDAY 6 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

[Turn over

Section A (36 marks)

1 In a survey, a sample of 44 fields is selected. Their areas (x hectares) are summarised in the grouped frequency table.

Area (x)	$0 < x \leqslant 3$	$3 < x \leqslant 5$	$5 < x \leqslant 7$	$7 < x \leqslant 10$	$10 < x \leqslant 20$
Frequency	3	8	13	14	6

(i) Calculate an estimate of the sample mean and the sample standard deviation. [4]

(ii) Determine whether there could be any outliers at the upper end of the distribution. [2]

2 In the 2001 census, people living in Wales were asked whether or not they could speak Welsh. A resident of Wales is selected at random.

- W is the event that this person speaks Welsh.
- C is the event that this person is a child.

You are given that $\mathrm{P}(W) = 0.20$, $\mathrm{P}(C) = 0.17$ and $\mathrm{P}(W \cap C) = 0.06$.

(i) Determine whether the events W and C are independent. [2]

(ii) Draw a Venn diagram, showing the events W and C, and fill in the probability corresponding to each region of your diagram. [3]

(iii) Find $\mathrm{P}(W \mid C)$. [2]

(iv) Given that $\mathrm{P}(W \mid C') = 0.169$, use this information and your answer to part **(iii)** to comment very briefly on how the ability to speak Welsh differs between children and adults. [1]

3 In a game of darts, a player throws three darts. Let X represent the number of darts which hit the bull's-eye. The probability distribution of X is shown in the table.

r	0	1	2	3
$\mathrm{P}(X = r)$	0.5	0.35	p	q

(i) (*A*) Show that $p + q = 0.15$. [1]

(*B*) Given that the expectation of X is 0.67, show that $2p + 3q = 0.32$. [1]

(*C*) Find the values of p and q. [2]

(ii) Find the variance of X. [3]

4 A small business has 8 workers. On a given day, the probability that any particular worker is off sick is 0.05, independently of the other workers.

(i) A day is selected at random. Find the probability that

(A) no workers are off sick, [2]

(B) more than one worker is off sick. [3]

(ii) There are 250 working days in a year. Find the expected number of days in the year on which more than one worker is off sick. [2]

5 A psychology student is investigating memory. In an experiment, volunteers are given 30 seconds to try to memorise a number of items. The items are then removed and the volunteers have to try to name all of them. It has been found that the probability that a volunteer names all of the items is 0.35. The student believes that this probability may be increased if the volunteers listen to the same piece of music while memorising the items and while trying to name them.

The student selects 15 volunteers at random to do the experiment while listening to music. Of these volunteers, 8 name all of the items.

(i) Write down suitable hypotheses for a test to determine whether there is any evidence to support the student's belief, giving a reason for your choice of alternative hypothesis. [4]

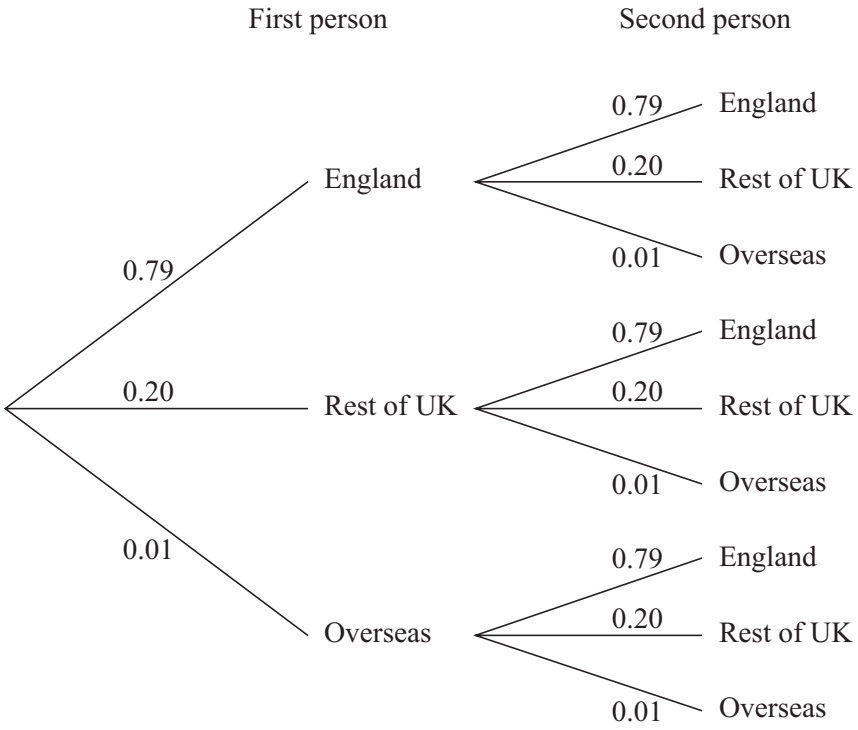
(ii) Carry out the test at the 5% significance level. [4]

[Turn over

Section B (36 marks)

6 In a large town, 79% of the population were born in England, 20% in the rest of the UK and the remaining 1% overseas. Two people are selected at random.

You may use the tree diagram below in answering this question.



(i) Find the probability that

(*A*) both of these people were born in the rest of the UK, [2]

(*B*) at least one of these people was born in England, [3]

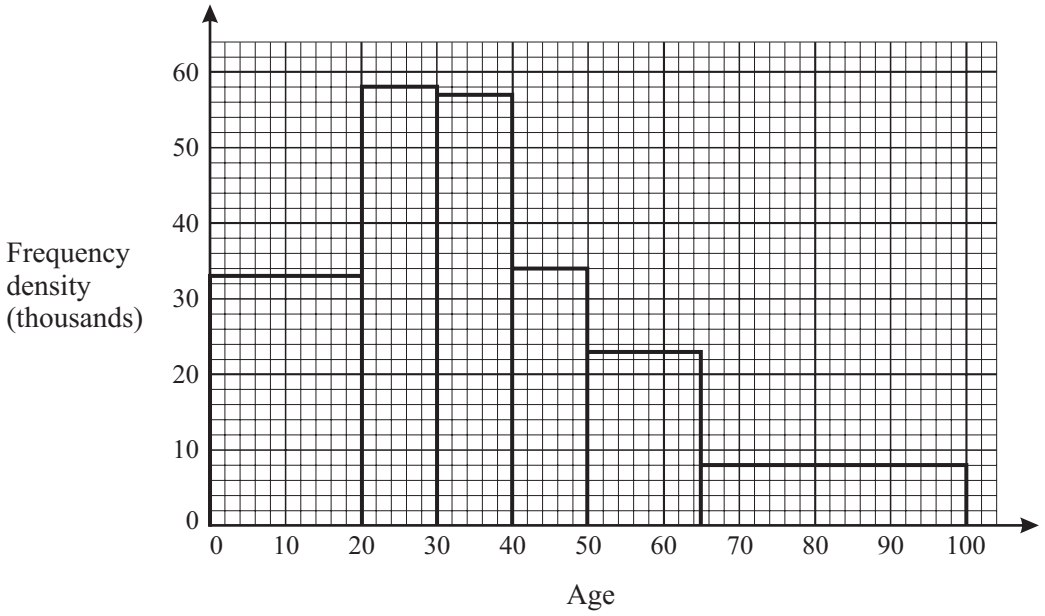
(*C*) neither of these people was born overseas. [2]

(ii) Find the probability that both of these people were born in the rest of the UK given that neither was born overseas. [3]

(iii) (*A*) Five people are selected at random. Find the probability that at least one of them was not born in England. [3]

(*B*) An interviewer selects n people at random. The interviewer wishes to ensure that the probability that at least one of them was not born in England is more than 90%. Find the least possible value of n. You must show working to justify your answer. [3]

7 The histogram shows the age distribution of people living in Inner London in 2001.



Data sourced from the 2001 Census, www.statistics.gov.uk

(i) State the type of skewness shown by the distribution. [1]

(ii) Use the histogram to estimate the number of people aged under 25. [3]

(iii) The table below shows the cumulative frequency distribution.

Age	20	30	40	50	65	100
Cumulative frequency (thousands)	660	1240	1810	a	2490	2770

(*A*) Use the histogram to find the value of a. [2]

(*B*) Use the table to calculate an estimate of the median age of these people. [3]

The ages of people living in Outer London in 2001 are summarised below.

Age (x years)	$0 \leqslant x < 20$	$20 \leqslant x < 30$	$30 \leqslant x < 40$	$40 \leqslant x < 50$	$50 \leqslant x < 65$	$65 \leqslant x < 100$
Frequency (thousands)	1120	650	770	590	680	610

(iv) Illustrate these data by means of a histogram. [5]

(v) Make two brief comments on the differences between the age distributions of the populations of Inner London and Outer London. [2]

(vi) The data given in the table for Outer London are used to calculate the following estimates.

Mean 38.5, median 35.7, midrange 50, standard deviation 23.7, interquartile range 34.4.

The final group in the table assumes that the maximum age of any resident is 100 years. These estimates are to be recalculated, based on a maximum age of 105, rather than 100. For each of the five estimates, state whether it would increase, decrease or be unchanged. [4]

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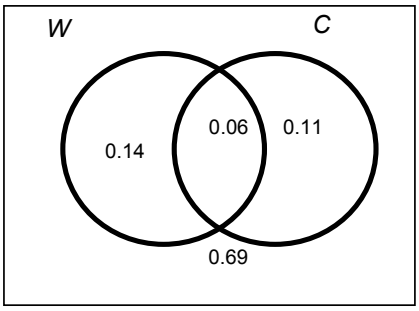
Acknowledgements:

Q.2 & Q.7 Data sourced from the 2001 Census, www.statistics.gov.uk

4766 Statistics 1

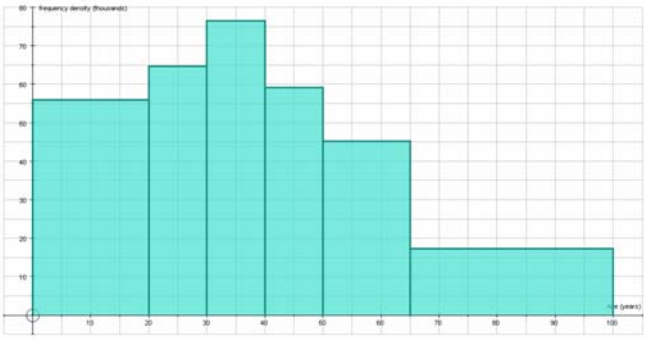
Q1 (i)	Mean = 7.35 (or better) Standard deviation: 3.69 – 3.70 (awfw) Allow s^2 = 13.62 to 13.68 Allow rmsd = 3.64 – 3.66 (awfw) **After B0, B0 scored** then if at least 4 correct mid-points seen or used.{**1.5, 4, 6, 8.5, 15}** Attempt of their mean = $\frac{\sum fx}{\mathbf{44}}$, with $301 \leq$ fx $\leq$ 346 and fx strictly from mid-points not class widths or top/lower boundaries.	B2cao $\sum fx$ = 323.5 B2cao $\sum fx^2$ = 2964.25 (B1) for variance s.o.i.o (B1) for rmsd (B1) mid-points (B1) $6.84 \leq$ mean ≤ 7.86	**4**
(ii)	Upper limit = 7.35 + 2 × 3.69 = 14.73 or 'their sensible mean' + 2 × 'their sensible s.d.' So there could be one or more outliers	M1 (with s.d. < mean) E1**dep** on B2, B2 earned and comment	**2**
		TOTAL	**6**
Q2 (i)	P(W) × P(C) = 0.20 × 0.17 = 0.034 P($W \cap C$) = 0.06 (given in the question) Not equal so not independent (Allow 0.20 × 0.17 ≠ 0.06 or ≠ p (W ∩ C) so not independent).	M1 for multiplying or 0.034 seen A1 (numerical justification needed)	**2**
(ii)	 The last two G marks are independent of the labels	G1 for two **overlapping** circles labelled G1 for 0.06 and either 0.14 or 0.11 in the **correct places** G1 for all 4 **correct** probs in the **correct** places **(including the 0.69)** NB No credit for Karnaugh maps here	**3**
(iii)	P($W \mid C$) = $\frac{\mathrm{P}(W \cap C)}{\mathrm{P}(C)} = \frac{0.06}{0.17} = \frac{6}{17} = 0.353$ (awrt 0.35)	M1 for 0.06 / 0.17 A1 cao	~~**2**~~

(iv)	Children are more likely than adults to be able to speak Welsh or 'proportionally more children speak Welsh than adults' Do not accept: **'**more Welsh children speak Welsh than adults'	E1FT Once the correct idea is seen, apply ISW	**1**
		TOTAL	**8**
Q3 (i)	$(\boldsymbol{A})$ $0.5 + 0.35 + \boldsymbol{p} + \boldsymbol{q} = 1$ so $\boldsymbol{p} + \boldsymbol{q} = 0.15$ $(\boldsymbol{B})$ $0\times0.5 + 1\times0.35 + 2\boldsymbol{p} + 3\boldsymbol{q} = 0.67$ so $2\boldsymbol{p} + 3\boldsymbol{q} = 0.32$ $(\boldsymbol{C})$ from above $2\boldsymbol{p} + 2\boldsymbol{q} = 0.30$ so $\boldsymbol{q} = 0.02, \boldsymbol{p} = 0.13$	B1 p + q in a correct equation before they reach p + q =0.15 B1 2p + 3q in a correct equation before they reach 2p + 3q = 0.32 (B1) for any 1 correct answer B2 for both correct answers	**1** **1** **2**
(ii)	E(X^2) = $0\times0.5 + 1\times0.35 + 4\times0.13 + 9\times0.02 = 1.05$ Var(X) = 'their 1.05' – 0.67^2 = 0.6011 (awrt 0.6) (M1, M1 can be earned with their p^+ and q^+ but not A mark)	M1 $\Sigma x^2 p$ (at least 2 non zero terms correct) M1dep for (– 0.67²), provided Var(X) > 0 A1 cao (No n or n-1 divisors)	**3**
		TOTAL	**7**
Q4 (i)	X ~ B(8, 0.05) $(\boldsymbol{A})$ $P(\boldsymbol{X} = 0) = 0.95^8 = 0.6634$ 0.663 or better *Or* using tables $P(\boldsymbol{X} = 0) = 0.6634$ $(\boldsymbol{B})$ $P(\boldsymbol{X} = 1) = \binom{8}{1} \times 0.05 \times 0.95^7 = 0.2793$ $P(\boldsymbol{X} > 1) = 1 - (0.6634 + 0.2793) = 0.0573$ *Or* using tables P(X > 1) = 1 – 0.9428 = 0.0572	M1 0.95^8 A1 CAO Or B2 (tables) M1 for P(X = 1)(allow 0.28 or better) M1 for 1 – P(X ≤ 1) must have both probabilities A1cao (0.0572 – 0.0573) M1 for P(X ≤ 1) 0.9428 M1 for 1 – P(X ≤ 1) A1 cao (must end in…2)	**2** **3**
(ii)	Expected number of days = 250×0.0572 = 14.3 awrt	M1 for 250 x prob(B) A1 FT but no rounding at end	**2**
		TOTAL	**7**

Q5 (i)	Let p = probability of remembering or naming all items (for population) (whilst listening to music.) H_0: $p = 0.35$ H_1: $p > 0.35$ H_1 has this form since the student believes that the probability will be increased/ improved/ got better /gone up.	B1 for definition of p B1 for H_0 B1 for H_1 E1dep on p>0.35 in H_0 In words not just because p > 0.35	**4**
(ii)	Let $X \sim B(15, 0.35)$ ***Either:*** $P(X \geq 8) = 1 - 0.8868 = 0.1132 > 5\%$ Or $0.8868 < 95\%$ So not enough evidence to reject H_0 (Accept H_o) Conclude that there is not enough evidence to indicate that the probability of remembering all of the items is improved / improved/ got better /gone up. (when listening to music.) --- ***Or:*** Critical region for the test is {9,10,11,12,13,14,15} 8 does not lie in the critical region. So not enough evidence to reject H_0 Conclude that there is not enough evidence to indicate that the probability of remembering all of the items is improved / improved/ got better /gone up. (when listening to music.) --- ***Or*:** The smallest critical region that 8 could fall into is {8, 9, 10, 11, 12, 13, 14, and 15}. The size of this region is 0.1132 0.1132 > 5% So not enough evidence to reject H_0 Conclude that there is not enough evidence to indicate that the probability of remembering all of the items is improved (when listening to music)	***Either:*** M1 for probability (0.1132) M1**dep** for comparison A1**dep** E1**dep on all previous marks** for conclusion in **context** --- ***Or:*** M1 for correct CR(no omissions or additions) M1**dep** for 8 does not lie in CR A1**dep** E1**dep on all previous marks** for conclusion in **context** --- **Or**: M1 for CR{8,9,…15}**and** size = 0.1132 M1 **dep** for comparison A1**dep** E1**dep on all previous marks** for conclusion in **context**	**4**
		TOTAL	**8**

	Section B		
Q6 (i)	(*A*) P(both rest of UK) = 0.20×0.20 = 0.04	M1 for multiplying A1cao	**2**
	(*B*) *Either:* **All 5 case** P(at least one England) = (0.79 x 0.20) + (0.79 x 0.01) + (0.20 x 0.79) + (0.01 x 0.79) + (0.79 x 0.79) = 0.158 + 0.0079 + 0.158 + 0.0079 + 0.6241 = 0.9559 *Or* P(at least one England) = 1 – P(neither England) = 1 – (0.21 $\times$ 0.21) = 1 – 0.0441 = 0.9559 or listing all = 1 –{ (0.2 x 0.2) + (0.2 x 0.01) + (0.01 x 0.20) + (0.01x 0.01)} = 1 – (**) = 1 – { 0.04 + 0.002 + 0.002 + 0.0001) = 1 – 0.0441 = 0.9559 Or: **All 3 case** P(at least one England) = = 0.79 $\times$ 0.21 + 0.21 $\times$ 0.79 + 0.79^2 = 0.1659 + 0.1659 + 0.6241 = 0.9559	M1 for any correct term (3case or 5case) M1 for correct sum of all 3 (or of all 5) with **no extras** A1cao (condone 0.96 www) *Or* M1 for 0.21 $\times$ 0.21 or for (**) fully enumerated or 0.0441 seen M1**dep** for 1 – (1st part) A1cao See above for 3 case	**3**
	(*C*)*Either* 0.79 x 0.79 + 0.79 x 0.2 + 0.2 x 0.79 + 0.2 x 0.2 = 0.9801 *Or* 0.99 $\times$ 0.99 = 0.9801 Or 1 – { 0.79 x 0.01 + 0.2 x 0.01 + 0.01 x 0.79 + 0.01 x 0.02 + 0.01^2 } = 1 – 0.0199 = 0.9801	M1 for sight of all 4 correct terms summed A1 cao (condone 0.98 www) *or* M1 for 0.99 x 0.99 A1cao ***Or*** M1 for everything 1 – {…..} A1cao	**2**
(ii)	P(both the rest of the UK | neither overseas) $= \frac{\text{P(the rest of the UK } \textit{and} \text{ neither overseas)}}{\text{P(neither overseas)}}$ $= \frac{0.04}{0.9801} = 0.0408$ {Watch for: $\frac{answer(A)}{answer(C)}$ as evidence of method (p <1)}	M1 for numerator of 0.04 or 'their answer to (i)(A)' M1 for denominator of 0.9801 or 'their answer to (i) (C)' A1 FT (0 < p < 1) 0.041 at least	**3**

(iii)	(*A*) Probability = $1 - 0.79^5$ $= 1 - 0.3077$ $= 0.6923$ (accept awrt 0.69) see additional notes for alternative solution	M1 for 0.79^5 or 0.3077... M1 for $1 - 0.79^5$ dep A1 CAO	
	(*B*) $1 - 0.79^n > 0.9$ **EITHER:** $1 - 0.79^n > 0.9$ or $0.79^n < 0.1$ (condone = and $\geq$ throughout) but not reverse inequality $n > \frac{\log 0.1}{\log 0.79}$, so $n > 9.768\ldots$ Minimum $n = 10$ Accept $n \geq 10$	M1 for equation/inequality in n (accept either statement opposite) M1(indep) for process of using logs i.e. $\frac{\log a}{\log b}$ A1 CAO	**3**
	--- **OR** (using trial and improvement): Trial with 0.79^9 or 0.79^{10} $1 - 0.79^9 = 0.8801$ (< 0.9) or $0.79^9 = 0.1198$ (> 0.1) $1 - 0.79^{10} = 0.9053$ (> 0.9) or $0.79^{10} = 0.09468$ (< 0.1) Minimum $n = 10$ Accept $n \geq 10$ --- NOTE: $n = 10$ unsupported scores SC1 only	--- M1(indep) for sight of 0.8801 **or** 0.1198 M1(indep) for sight of 0.9053 **or** 0.09468 A1 dep **on both M's** cao ---	**3**
		TOTAL	**16**

Q7 (i)	Positive	B1	**1**
(ii)	Number of people = 20×33 (000) + 5×58 (000) = 660 (000) + 290 (000) = 950 000	M1 first term M1(indep) second term A1 cao NB answer of 950 scores M2A0	**3**
(iii)	(*A*) *a* = 1810 + 340 = 2150 (*B*) Median = age of 1 385 (000^{th}) person or 1385.5 (000) Age 30, cf = 1 240 (000); age 40, cf = 1 810 (000) Estimate median = (30) + $\frac{145}{570} \times 10$ Median = 32.5 years (32.54...) If no working shown then 32.54 or better is needed to gain the M1A1. If 32.5 seen with **no previous working** allow SC1	M1 for sum A1 cao 2150 or 2150 thousand but not 215000 B1 for 1 385 (000) or 1385.5 M1 for attempt to interpolate $\frac{145k}{570k} \times 10$ (2.54 or better suggests this) A1 cao min 1dp	**2** **3**
(iv)	Frequency densities: 56, 65, 77, 59, 45, 17 *(accept 45.33 and 17.43 for 45 and 17)* 	B1 for any one correct B1 for all correct (soi by listing or from histogram) **Note: all G marks below *dep* on attempt at frequency density, NOT frequency** G1 Linear scales on both axes (no inequalities) G1 Heights FT their listed fds or all must be correct. **Also widths. All blocks joined** G1 Appropriate label for vertical scale eg 'Frequency density (thousands)', 'frequency (thousands) per 10 years', 'thousands of people per 10 years'. (allow key). **OR f.d.**	**5**

(v)	Any two suitable comments such as: Outer London has a greater proportion (or %) of people under 20 (or almost equal proportion) The modal group in Inner London is 20-30 but in Outer London it is 30-40 Outer London has a greater proportion (14%) of aged 65+ **All** populations in **each** age group are higher in Outer London Outer London has a more evenly spread distribution or balanced distribution (ages) o.e.	E1 E1	**2**
(vi)	Mean increase ↑ median unchanged (-) midrange increase ↑ standard deviation increase ↑ interquartile range unchanged. (-)	Any one correct B1 Any two correct B2 Any three correct B3 All **five** correct B4	**4**
		TOTAL	**20**